Ansatz-Free Learning of Lindbladian Dynamics In Situ

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SCAN ME

Motivation

- Characterizing open quantum dynamics is critical for calibrating hardware and designing error correction.
- The Lindblad equation captures both coherent and dissipative dynamics under Markovian noise.
- Prior protocols assume known interaction structure — restrictive when the relevant error mechanisms are unknown.

Can we efficiently learn the full Lindbladian from its time evolution in situ, without structural assumptions?

Lindbladian Model

$$\frac{d}{dt}\rho = \mathscr{L}(\rho) = -i\sum_{P_i\in\mathcal{S}_H} h_i[P_i,\rho] + \sum_{P_i,P_j\in\mathcal{S}_D} a_{ij}\left(P_i\rho P_j - \frac{1}{2}\{P_jP_i,\rho\}\right)$$

- P_i : n-qubit Pauli operators
- $\mathcal{S}_H, \mathcal{S}_D$: unknown Hamiltonian and dissipator supports
- M = total nonzero Lindbladian terms (or an upper bound)
- η = smallest nonzero coefficient (or a lower bound)

Stage 1: Structure Learning

At short times, $e^{\mathscr{L}t}$ is approximated by its Taylor expansion with sparse Pauli structure. The derivatives of Pauli error rates directly reveal the generator structure:

$$\chi_{ii}^{(1)} = a_{ii} \text{ if } a_{ii} > 0 \quad \text{and} \quad \chi_{ii}^{(1)} = 0,\ \chi_{ii}^{(2)} \geq 2h_i^2 \text{ if } a_{ii} = 0$$

(1) Estimate all Pauli error rates $\chi_{ii}(t_m)$ at short times via population recovery [Flammia, O'Donnell, 2021].

(2) Fit Chebyshev interpolants; extract first and second derivatives at $t = 0$.

(3) Classify dissipator: $\chi_{ii}^{(1)} > \eta/2 \implies \widehat{\mathcal{S}}_D \leftarrow P_i$

Classify Hamiltonian: $\chi_{ii}^{(2)} \geq \eta^2 \implies \widehat{\mathcal{S}}_H \leftarrow P_i$

(4) Output candidate supports $\widehat{\mathcal{S}}_D$ and $\widehat{\mathcal{S}}_H \leftarrow \widehat{\mathcal{S}}_H \cup \widehat{\mathcal{S}}_D$

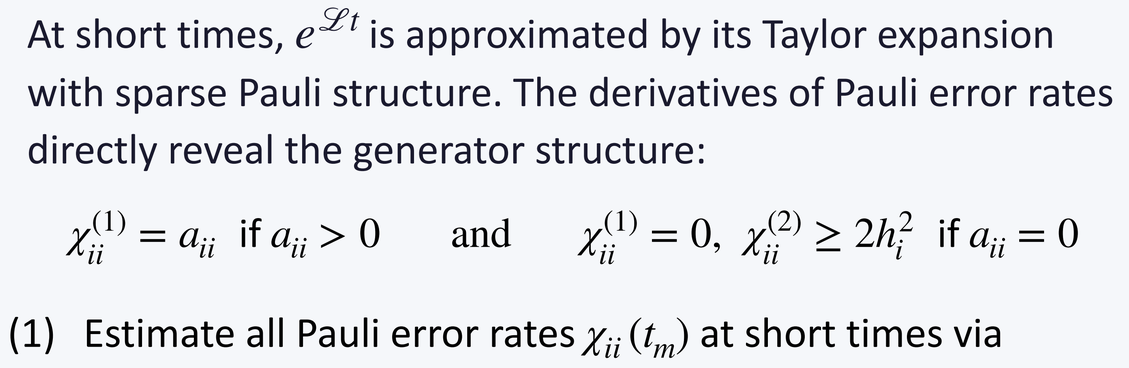

Figure 1. (a) Population recovery, (b) Pauli error rate distributions, (c) Chebyshev derivative fitting separates Hamiltonian ($\propto t^2$) from dissipator ($\propto t$), (d) recovered supports.

Result 1 — Structure Learning

We can recover a polynomially-sized superset of the Lindbladian structure using $\tilde{\mathcal{O}}\left(M^4/\eta^4\right)$ queries to $e^{\mathscr{L}t}$ with time resolution $t = \tilde{\Theta}(1/M)$.

Stage 2: Coefficient Learning

Given candidate supports $\widehat{\mathcal{S}}_D$ and $\widehat{\mathcal{S}}_H$, estimate all Lindbladian coefficients $\{h_i, a_{ij}\}$ to additive accuracy ε.

Linear response relation: $$\frac{d}{dt}\langle O(t)\rangle\Big|_{t=0} = \mathrm{tr}\left(\rho\,\mathscr{L}^\dagger(O)\right)$$

(1) Select Pauli probes (ρ, O) via patchwise tomography on candidate supports.

(2) For each probe, sample $f(t) = \mathrm{tr}\{\rho\, e^{t\mathscr{L}^\dagger}(O)\}$ at Gauss–Chebyshev nodes; fit and extract df/dt at $t = 0$.

(3) Assemble the linear system $\mathbf{d} = C\mathbf{x}$ from the derivative estimates.

(4) Solve classically; error amplification controlled by $\nu = \|C^{-1}\|_{\infty\to\infty}$

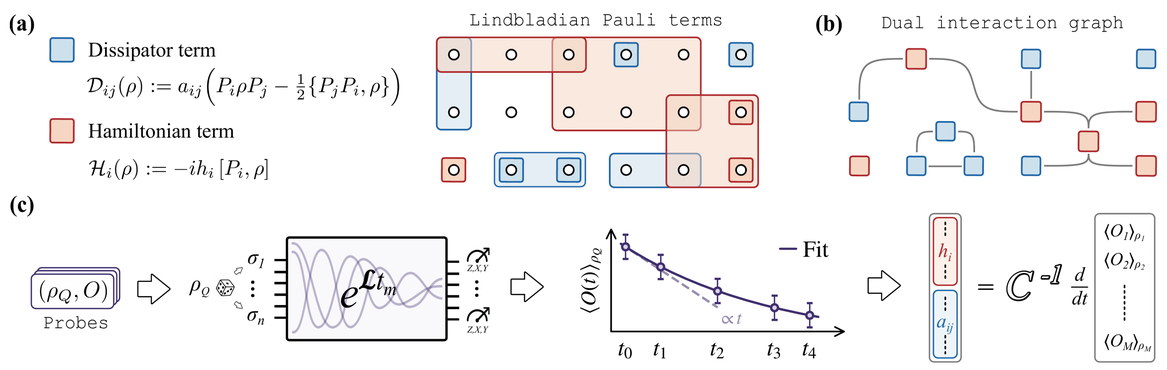

Figure 2. (a) Lindbladian Pauli terms, (b) dual interaction graph; degree $\mathfrak{d}$ controls the derivatives of Pauli observables, (c) linear system from short-time derivatives via Chebyshev interpolation.

Result 2 — Coefficient Learning

Given a superset of the Lindbladian structure, we can recover all coefficients $\{h_i, a_{ij}\}$ to ε-accuracy using

$$\tilde{\mathcal{O}}\left(\min(9^k, \widehat{M})\,\mathfrak{d}^2\nu^2/\varepsilon^2\right)$$

queries to $e^{\mathscr{L}t}$ with time resolution $t = \tilde{\Theta}(1/\mathfrak{d})$.

Time Resolution

- Our protocol uses minimum evolution time $t_{\min} = \tilde{\Theta}(1/M)$.
- This is provably near-optimal. Coarser resolution forces any protocol to use exponentially many samples. Key idea: rapid mixing drives all input states exponentially close to the maximally mixed state, erasing structural information.

Result 3 — Optimality of Time Resolution

At resolution $t_0 = \Theta\left(M^{2/\kappa-1}\right),\ \kappa \geq 2$ there exist M-sparse Lindbladians requiring $\Omega\left(e^n\right)$ queries.

Takeaways

- First ansatz-free Lindbladian learning protocol in situ.
- No prior structure, no ancillas — product-state preparations and Pauli measurements only.
- Total sample complexity $m_{\text{tot}} = \tilde{\mathcal{O}}\left(\frac{M^4\nu^2}{\varepsilon^4}\right)$

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